QUAN 2101, Fall 2023 Ranger Liu
Unit 4: Quantum Romantics

Lecture I: Foundations of Quantum Romantics - Transcript

(00:00) I. Introduction, pp. 4 – 5

Hello everyone. Today we're going to be starting a new unit, Unit 4, on Quantum Romantics. In the notebooks that I just passed out, you'll find the lecture notes for this week's lecture, which you can follow along with today. You'll also find this week's problem set which is due on Thursday, so keep that in mind. For now, let's turn to page four, which is the introduction.

We'll start with a brief road map of where we've been and where we're going today. In August, we started with Classical Mechanics as our first unit, and then we moved into Quantum Mechanics as our second unit. We were treating Quantum Mechanics as a more accurate extension of Classical Mechanics. Then for our third unit, we went back into the classical to look at Classical Romantics, and now we're looking into Quantum Romantics - again, as a more accurate extension of Classical Romantics.

Now, both of these fields, Romantics and Mechanics, are subfields of the greater umbrella of physics. And, as we previously discussed, physics is just our scientific and methodical attempt at understanding, modeling, and eventually predicting behaviors of the universe. So in mechanics, we studied behaviors of force and motion in particles, and in romantics, we're studying the behavior of love and desire in persons. And now, in both fields we're modeling these behaviors with mathematics and theories, with the eventual goal of using these models to predict these behaviors.

Now, in moving from Classical Romantics to Quantum Romantics, we're simply applying a different set of mathematics and theories to model and predict this behavior. And in particular, we're applying the same framework that we used to apply to Quantum Mechanics, so today in this lecture we'll be referring a lot back to Quantum Mechanics in order to learn more about Quantum Romantics. So now let's flip back to page one for a moment, just to get a sense of the rest of today's lecture. I've also written up the schedule here, on the side.

So, we finished with our introduction, and in the second part, it's going to be our conceptual overview where we'll make the foundational analogy as I mentioned before, from Quantum Mechanics to Quantum Romantics, specifically relating the introduction of spin, quantum mechanical spin, to the introduction of emotion and emot states in Quantum Romantics.

After that, we'll also touch back on ontological causality, briefly, as it relates to both Quantum Mechanics and Quantum Romantics. You'll remember this as that ongoing philosophical debate, over the reason why physical reality is quantized and discrete and finite. There's some really interesting extensions of that debate into the realm of quantum [romantics] that I think is really important for you to know. So even though that's not the focus of this class, we'll be talking about it, a little bit.

Then, in part three we're going to go over the mathematics of Quantum Romantics in more detail. We're going to start by looking at emot state bases, and look at how we can represent emot states using Dirac notation and Heisenberg's matrix mechanics, as we did in Quantum Mechanics to represent spin states.

Then, in part four, we're going to discuss Love Questions, LQs, which are our Quantum Romantic equivalent to Quantum Mechanical operators. You'll remember from Quantum Mechanics that our operators are our mathematical representations of measurable quantities and the act of measuring them. So similarly, our LQs are modeling our Quantum Romantical methods of measuring the emot states of persons. Now, LQs are what really make Quantum Romantics different from Quantum Mechanics, and they sort of directly lead into the two principal paradoxes of Quantum Romantics.

So the first paradox is the Principal Paradox of Interrogative Necessity, and this deals with the sort of fact that you need to ask a question in order to measure a state, but asking that question inevitably changes the state. And so how do you deal with that, mathematically? That's what this paradox is concerned about.

The second paradox is the Paradox of Subjectivity which deals with the uniquely Quantum Romantical phenomenon that the effect of an applied LQ depends not ONLY on the person it's applied to, but also on the person who is applying it. There's a level of subjectivity in Quantum Romantics that we don't see at all in Quantum Mechanics, and so these two paradoxes right are really the things that are separating Quantum Romantics and Quantum Mechanics even though, as we'll see, they share a lot of the same mathematical foundational principles.

So, at the end of this first lecture, you'll be equipped with these mathematical foundations as well as these theoretical foundations of Quantum Romantics. In the problem set due Thursday, I will be asking you to engage with these concepts and apply these concepts and think about them a little bit more critically.

II. Conceptual Overview

(06:49) 2.1 QM Review, pp. 6 – 7

All right. So, let's get into it. We're going to start with our conceptual overview now.

So, as I mentioned, we're going to start with the analogy to Quantum Mechanics. To do that, we're going to do a bit of a review of Quantum Mechanics. So we're going to start on this side, we can turn to the next page, after the Introduction, to Part Two, conceptual overview, and this is going to be QM Review.

So when we started learning about Quantum Mechanics, we started by talking about spin, which is a quantum mechanical property inherent to a particle. And we talked about how spin is related to the classical notion of a magnetic moment, which will be represented as mu in the vector form. And so this is important to note, that spin and magnetic moment are related concepts, they're mathematically and physically related concepts, but they are different, distinct concepts. Spin comes uniquely from a sort of quantum mechanical principle, so just keep that in mind.

So in our classical model, we modeled magnetic moment mu in the z direction by the equation magnitude of mu * cosine (theta). And what this really tells us is that, in the classical framework, that mu of z can take any one of those infinite possible values, infinite number of possible values, between the continuous range, negative mu to positive mu. So, in the classical sense, right, mu z should be both infinite and continuous, which will notate like this.

Now, what we find when we do the experiment, of course, is that mu z is actually only measured to be two discrete values. It's going to be either plus or minus h bar over two, which in particular are values of the magnetic moment that we associate with spin-1/2 particles such as the electron. And so, in quantum mechanics, which is the more accurate representation of reality, we find that things are discrete. They can only have discrete values, and in particular they only have finite possibilities of discrete values to take. And so we'll represent that symbolically, by drawing a circle for finiteness and then four dots around it to show discretization.

So, it's important to note this: in the classical framework, things are infinite and continuous; in the quantum framework, things are finite and discrete.

Now of course, in quantum [mechanics] we're not just talking about spin. We also talk about position and momentum, for example, X and P, as well as a whole host of other things that we might measure. It's important to note about position and momentum, of course, that they also exist in the classical framework, X and P. And these two sets of concepts here are actually the exact same concept, they're just modeled differently in these two different frameworks. Whereas spin and magnetic moment are related, but different, these two are the same, but modeled separately in classical and quantum. In particular, in the classical, X and P are also given infinite possible values within continuous ranges, and in the quantum sense X and P are shown actually to only take a finite number of possible discrete values.

And then the last thing that we remember about X and P is that they are canonically conjugate, meaning that they display Heisenberg's Uncertainty Principle: you cannot know one exactly while also knowing the other exactly, at the same time, which we write out mathematically as change in X times change in P must be greater than or equal to h bar over two.

(12:18) 2.2 QM to QR Analogy, pp. 8 – 9

So now, let's turn the page to the next section which is our QM to QR Analogy. And, before we make this analogy, I just want to specify what we're thinking about in these two cases. In quantum mechanics, or any mechanics, we're talking about particles. In the romantics, we're talking about persons. And by person, I mean the encapsulation of the individual as this conceptual model responding to romantic interactions from love and desire.

So, in quantum mechanics, just as we started with spin, in quantum romantics, we're going to start with the concept of emotion. And emotion is going to be related to our classical romantic concept of feeling. And again, this relation here is going to be similar to the relation between magnetic moment and spin in that they are two separate, distinct concepts that are physically and mathematically related. And we'll remember from our study of classical romantics that feeling is similarly infinite and continuous in possible values. And what we'll find in quantum romantics is that emotion is going to be finite and discrete in possible values.

What we're also going to study in quantum romantics a little bit later is status. And you might remember from classical romantics that this also exists in that framework, and this time these two are going to be the same concepts, in the same sense that position and momentum are the same concepts in classical and quantum, represented differently. Status in classical romantics is infinite and continuous, and then again down here in quantum romantics, what we'll see is that it's finite and discrete.

Lastly, for our canonically conjugate pair of interest in the quantum romantic side, we actually find that the pair in question is going to be emotion and status, which is a little bit different than our analogy. So these are going to be our canonically conjugate pair, and the equation for this will be change in s, change in status, times change in e, change in emotion, is going to be greater than or equal to heart bar over two. And this is going to be our quantum romantical constant.

So for today's class, we're going to be mainly focusing on emotion and emot states in relationship to spin states. We're going to talk about status and the relationship between emotion and status in the next coming weeks. For today, we're going to be focusing on emotion.

(15:42) 2.3 Ontological Causality in QM, pp. 10 – 11

So if you turn the page the next two following pages are going to be the ontological causality discussion in both QM and QR. And so this is going to be the last brief tangent before we jump directly into the math.

So we'll start with the quantum mechanical first. You'll remember that the question being posed by ontological causality in quantum mechanics is: Why is reality - meaning, physical reality as we see it in quantum mechanics - both finite and discrete? Why is that? Trying to get at the question of the nature of particles and why reality is the way that it is.

So, as we discussed, the first camp of opinion in this field is of Discretization by Origin, which holds that, well, particles are just discrete and finite by nature. Inherently, they are just discrete and finite. And so when they externalize these observables that we can then measure, when they produce these observables, they are obviously going to be discrete and finite just because the particles themselves are finite and discrete. Discretization by Origin: the nature of the thing is present in the production - in things that are produced by that thing.

The second camp is of Discretization by Poiesis. Poiesis: p-o-i-e-s-i-s, which is a Greek term meaning making – process of making – and this camp holds that particles exist in some kind of infinite and continuous way. The terminology and the symbols that they use, they say that particles exist in some "infinite firmament" that is also continuous. And that's the symbology for that, that they hold to. And so the idea here is that particles are not inherently discrete and finite; they are inherently infinite and continuous. However, when they externalize into the observables that we're able to measure, it's this process here of externalization, the process of poiesis, of making, that is actually a discretizing process. And so it's by process that reality becomes discrete and finite.

Now of course, this second theory is not widely held, the first one is the much more common position to take. In particular, the second one seems to give particles almost a sense of quasi-consciousness that's kind of strange for scientists to agree to. So the reason I bring this up is that, with the arrival of quantum romantics onto the scene, the application of ontological causality in the realm of quantum romantics then has reverberated back into the quantum mechanical scene to shake things up a little bit.

(19:22) 2.4 Ontological Causality in QR, pp. 12 – 13

And so if we look at the application in the quantum romantical side, the question now becomes: Why are romantic communications finite and discrete? Why is it that emotions and status are finite and discrete in this way? What is the cause of that?

And so the two camps now in the quantum romantics side is first Discretization by Origin, which now holds that people are somehow inherently finite and discrete. And so then of course, by their externalizations of motion, status, communication, that these things are all discrete and finite because people are finite and discrete.

And then the second theory, which is Discretization by Poiesis, says that people exist in some "infinite firmament" of continuity, and then the process of externalization into communication is inherently a discretizing process. So it's discretization by, by process of making.

Now, the second opinion here is much more widely held in the field of ontological causality as applied to quantum romantics, of course, because this theory seems to hold much better with people's lived experiences and personal experience. In particular, this infinite firmament of continuity is widely linked to the concept of consciousness, or also will sometimes be called soul. In the same way that emotion and feeling are our scientific terms that we give to describe properties, as opposed to discussing them in this more sort of spiritual or quotidian connotation - soul, meaning a very specific scientific term to represent this particular phenomenon. And so the second theory is much more widely accepted, in this realm, at least, and it's kind of a personal question, in my opinion. It's just: do you feel that your experience is kind of fundamentally unexpressable in its entirety? Which for me, personally, I think, is kind of true.

And so the sort of reverberation is, this second theory of Discretization by Poiesis, at least in the romantical side, is much more believable and comes with, at least, first person confirmation. And so then the idea is that, of course, if we as people have these sort of infinite firmaments of consciousness, of souls, and we externalize into communications that are incomplete, why is the same not true for particles?

Of course, it's an interesting question. There's wide debate about it. If you're interested, there's some stuff in the Supplemental Notes, and there's also a class that you can take on Quantum Semiotics that is explaining all this stuff in great, great detail. But for today at least, what I want you to know is that the mathematics and the formulation and discovery of these theories has greatly impacted the philosophical discussions going on around it. Xo it's important that you know it, at least, and that you know at the very least these two positions which are the most widely held positions.

(23:21) III. Emot State Bases, pp. 14 – 15

All right. I think that's enough theory and philosophy for this physics class. So, we're going to turn the page and now we're going to get into the mathematics proper. So, we're now on Section Three: Emot State Bases, and we're going to get into mathematics.

We're going to start with the Dirac notation and looking at bases, again with the same grounding in quantum mechanics.

So in quantum mechanics, when we first talked about bases, we talked about spin-½ particles. Again, our famous and beloved electron, it's a spin-½ particle. And in particular, spin-½ particles have two bases. We represented them initially as plus-z and minus-z. But if we write them in the s,m notation that we later learned, you'll also write them as (½, +½), and (½, -½). The spin value, measured value, spin value, measured value. And so these bases states represent the two measurements that are possible to be observed for a particular particle, for a spin-½ particle.

We also discussed spin-1 particles, which now have three bases states, three possible quantities that can be measured. We represented them as (1, -1), (1, 0), and (1, +1). Again, s,m notation: spin value, measured value, spin value, measured value, spin value, measured value. And then we also discussed that as you get higher up in spin, you get more and more basis states for those particles, meaning that certain particles can be measured in more and more discrete, separate states.

So, when we move over to the romantics side, we're looking instead at emotion and emotion states. So, for the emot-½ persons – people who are emot-½, as opposed to particles that are spin-½ – emot-½ persons also have just two bases, and we represent them notationally still as kets, but on the inside we'll draw a filled in heart and then a broken heart. And we call these states the LOVES ME state and the LOVES ME NOT state. These two are for an emot-½ person the only two measurable quantities.

Now when we talk about emot-1 persons, similarly they will also have three bases vectors. Curiously though, they still retain these two, the LOVES ME and LOVES ME NOT, and then the third one that's added on we'll represent notationally as a heart with a question mark, which we call IT'S COMPLICATED.

And so, already we're seeing a difference between romantics and mechanics. That LOVES ME and LOVES ME NOT in general, in higher emot states, are always going to be the two bases vectors, and then the remaining vectors are going to be just further shades of IT'S COMPLICATED. This IT'S COMPLICATED dimension just splits into finer and finer dimensions of complication.

And so, in this class today, we're going to be focusing on emot-1 particles, and we'll discuss the larger emot values later, and discuss how they deal with these shades of complication later. So the very last thing to talk about here is a concept called temporal mutability, which also is a little bit related to the ontological causality debate.

Temporal mutability, which is also sometimes called theory of change, you might have heard that. Essentially the idea is that, in quantum romantics, emot values are allowed to change. Persons can change their emot values over time, and so, if – here's a timeline, and at certain points in time, you measure a person's emot value, and it's, say, ½ at one day, and then it's 6 the other day, and then it's 3 another day, we still consider this to be the same person,

There's a certain amount of mutability that we grant to persons that we don't grant to particles in the quantum mechanical side. If you just think about, if we have a timeline and at certain times we measure different spin values, say ½, 6, and 3, we consider these to be all different particles. So there's a certain sense of individuality that we grant to persons that we don't necessarily grant to particles in our common scientific practice.

The connection there to the ontological causality – I'll just mention this because it's interesting – is that there's a vocal minority of the Discretization by Poiesis causalists who argue for particles to be granted this kind of temporal mutability so that particles can change their spin values but still be considered still the same sort of particle-soul stuff, as an individual. Very interesting, not particularly applicable to our study, but I just thought I'd mention it because it is, again, the sort of philosophical things that sprout off of these theoretical and mathematical constructions.

The important thing to remember is that persons can change their emot values over time. And in particular, this does actually happen quite often, which makes it very difficult to quantify, demographically, what emot values are more commonly seen.

IV. LQs (QR Operators)

(31:43) 4.1 Introduction to LQs, pp. 16 – 17

So, as I said, for the rest of today's lecture we're going to be focusing on the emot-1 person as our Illuminating example. So we can flip the page now, we're gonna start getting into the Love Questions, operators.

So for our emot-1 person, we can represent the general emot state just like we would in quantum mechanics, as a linear combination of the basis vectors. So for a general state, we'll just draw it as a heart unfilled. And so, it'll just be some coefficients multiplied by our basis vectors. And I'm going to write out the words here again, because I want us to notice something.

So this state here is representing a person's emot-1 state. You might call this person, "YOU", and so we'll put a little Y here to denote that this is the emot state of YOU, who is a emot-1 person. But then notice that, in describing this person's state, we're also invoking another person – a "ME", which somehow you can think of as the person who this YOU is expressing emotion towards, or has these emotions towards.

And so this ME really is the observer. Just keep this in mind here as we talk about the rest of today's lecture, because this is really at the heart of what makes quantum romantics very different from quantum mechanics. The observer is somehow invoked in the description of the observee.

So when we talk about the observer, right, we also want to talk about – how does an observer make measurements? So in quantum mechanics, just to go back for a brief moment, of course we use operators. Common operators being the [Hamiltonian] operator, time evolution, or spin operator, to measure spin.

In quantum romantics, as I mentioned at the beginning of class, we're going to be using LQs, Love Questions, which are the operators of quantum romantics. And so, I'll give you some of the more common LQs here. They often come in strong and weak pairs. So, the most common one is going to be strong-L and weak-L, which represent the LQs I LOVE YOU – strong-L, and then I LIKE YOU for weak-L. And these are also closely associated with the corresponding pair strong-Q and weak-Q, which represent DO YOU LOVE ME, and then weak-Q: DO YOU LIKE ME.

And note two things about these: first, that these are all verbal examples of LQs, and also note that they're not necessarily questions. I love you, I like you – these are not questions, and so just make a note of that, that "love questions" is a bit of a misnomer, because it doesn't have to be a question, it just has to be a certain communication from ME to YOU that tells ME about the emot state of YOU.

And so, I'll give you two more common LQs, this time non-verbal, again in a strong weak pair, strong-K, weak-K, which represent the action of a kiss, sort of continued, for strong K, and then the action of a kiss, a momentary kiss, peck on the cheek, perhaps. And these are going be examples of non-verbal LQs. So LQs come in all different shapes, different forms.

And then, how do you apply these LQs? Well, it's the same construction, mathematically, as in quantum mechanics. So we would write, for example, this expression here. And then this is just going to give me, the probability of measuring YOU, in the state LOVES ME, probability of finding YOU in the state LOVES ME through the use of this LQ here, strong-L, which is I LOVE YOU. So it's the probability of finding YOU in the state LOVES ME through I LOVE YOU. So you can see that the directionality here matters very much when we're talking about probabilities. It's finding YOU in state LOVES ME. That observer-observee invocation, it's very important.

(38:59) 4.2 The Principle Paradox of Interrogative Necessity, pp. 18 – 19

All right. So now, we'll turn the page again, and we're going to talk about the Principal Paradox of Interrogative Necessity, and we're going to go through that by looking at this equation here in a little bit more detail.

So if we were to try and evaluate this expression mathematically, we kind of have to go through three steps. First, take the YOU state, and then you apply the love question strong-L, and then collapse the state, the asked state, with respect to that particular basis vector, LOVES ME.

So conceptually, this first transformation here is an asking. This is the action of asking, using the love question to ask YOU. And then of course the asking demands an answering, here, and so this here is the statement of interrogative necessity: in order to measure the state of YOU, I must ask a question. There is no making a measurement without asking the question, and so that is the interrogative necessity.

What makes it paradoxical is, why we call it a paradox, is because of this thing right here. The asking will change the original state. These two things are not equivalent, the original state and the asked state are not equivalent. And so, in order to make a measurement, you must ask the question, but in asking the question, you change the thing that you're trying to measure. And thus that becomes a paradox.

And so, this first statement of the interrogative necessity principle is usually written as this: it says LQs are necessary for all measurements of emot states. To make a measurement, you must ask a question; there are no measurements made without the question.

And so, this conceptual sort of restriction, the requirement of the LQ, has a lot of mathematical ramifications. And one of these comes when we consider normalization. And so, on the quantum mechanical side again, if we look at the normalization, what we normally say is something like this: the probability of finding a particle in the state psi to be in the state psi is one. It's a sort of identity operator, of course a particle that's in a state, if you measure it, it has to always be in that state.

However, when we try to construct this mathematically in quantum romantics, of course we can write this down. But, if we look at this, it's of course missing the LQ. There's nothing in the middle, there. There's no question being asked in order to make this measurement, and thus this measurement is actually, experimentally, it's meaningless. It's experimentally impossible. This never happens in real life. And so although we can write this mathematically, what we say is that this is undefined. Because it could take all sorts of different values, and so this is the mathematical statement of the principle of interrogative necessity, as well.

And further, the way that we describe this with normalization is normally, this equation here gives us a requirement for normalization. It says, this equation always has to hold true for any state psi, so for any state psi, we must arrange its coefficients such that this equation, when we multiply them out, will always equal one.

Now that this is undefined in our quantum romantics side, we in general don't really need our emot states to be normalized; they can have more freedom, and so that is the second statement of the interrogative necessity principle: emot states need not be normalized.

And if you've been following what I've been saying so far, you might note that there's one more thing that we need, to make this all work out: if emot states are not normalized, then these probabilities that we get, how do we make them make sense? They could give us any values; they could be negative; they could be greater than one. How do we make it make sense?

So that is going to be the third statement in the principle of interrogative necessity. We need some normalization somewhere so that our probabilities can make sense. And so what we do, ultimately, to make this work out mathematically, this is a mathematical necessity, is we insert an extra step right in here, and usually we'll just say that before we do the final calculation of probability, we'll add a normalization constant. So, this would look like this: normalization constant n. And again, this is mathematically a necessity, but conceptually it does nothing at all.This is only required so that the numbers work out, so that the math works out.

And so, the way that we write this as the third statement of interrogative necessity is that we'll say: asked states must be normalized before calculating probability. And again, asked states, we just mean this state of the process right here, after it's been asked by the LQ.

All right. So that's the Principal Paradox of Interrogative Necessity. This is very, very important when you're working out your problems in your problem set, so make sure that you understand this really clearly. The normalization constant in particular can be a little tricky, but once you understand how it works, it's really just a constant so that everything works out for the probabilities to be less than one.

(47:02) 4.3 The Principle Paradox of Subjectivity, pp. 20 – 21

All right. So, we can flip the page now to the last page. We're going to continue considering this equation here, and delve more into the matrix mechanics. And in doing so we're also going to discuss the final Principal Paradox of Subjectivity.

All right. So we'll consider that same equation that I had before. Calculating the probability of finding YOU in the state LOVES ME through the love question I LOVE YOU. So how would we calculate this mathematically, numerically? How would we do this?

Well, if we were to go back to quantum mechanics, we would probably say, well, we can transform this into matrices, and then just multiply them out, get a numerical answer that way. So what that would look like, is that our bra here would probably turn into a row vector of some sort, the operator will turn into our square matrix, and then the ket will turn into a column matrix. Multiply out with matrix multiplication, and we get out a probability, p.

Now in general, this is perfectly fine to do in quantum romantics, and we just follow it through in the same way that you would imagine. So in particular, the basis vectors are all going to be represented as you would imagine. So we can just assign them in arbitrary but conventional order like so. And then of course since these are the kets, the bras will just be the complex transpose, so this will be 1 0 0. And then for the state vectors as well, we just represent them with the coefficients as you would in quantum mechanics. And then remember again from the principle of interrogative necessity that we must include a normalization constant in here. So we'll just include an N that will normalize the asked state before we take the probability.

So now the final question is, how do we represent the LQ in terms of a matrix? So, it's represented in the same way that we might represent operators in quantum mechanics. If you look in your notes, there should be a lovely large matrix in there. It's gonna have nine elements. And so I won't write it out for you today, because it's going to take quite some time, and it's nothing new that you haven't seen before, but I can write it out in its element wise representation, which is going to be like so: L, in the i,jth position, which is index, is going to be basis vector i, basis vector j, is the ket.

And so what we've done, of course, is turned one thing that we don't know how to represent numerically into nine things that we don't know how to represent numerically. And so the question still remains: How do we figure out what these things are numerically? And so the answer is that it's really hard to figure it out. It's really hard, you have to do it by experiment, and you have to do a lot of experiments, and in particular it's very, very difficult because of the Principle Paradox of Subjectivity.

And as I mentioned in the beginning, the Principle Paradox of Subjectivity simply states that LQs, when applied by different people, will have different results on the same YOU state. And so we can write this mathematically by saying, for instance, strong-L as applied from ME to YOU, on the emot state of YOU, this asked state will be different from the asked state that results from strong-L, the same strong-L, applied from an OTHER, other person O, to YOU, on the emot state of YOU.

So this is where the subjectivity comes in.When we're talking about defining LQs, you can't just define them by the conceptual representation of whether they represent I LOVE YOU, I LIKE YOU, DO YOU LOVE ME, DO YOU LIKE ME, all that stuff. You also have to represent them in the particular context of the person asking and the person answering.

So, diagrammatically, of course, we can also write this as so: for a YOU who is represented with an emot state like so, if we have a ME here and an OTHER here, we each have very different operator spaces of LQs. Mine will be from ME to YOU, and the OTHER's will be a set of LQs from O to Y, or OTHER to YOU, and these two will not be equivalent.

And so what this really tells us is that we actually need more notation in our states, in general. So for every love question we need to define not only who's asking it, but also who's answering it. So here we should write M to Y, ME to YOU, and in all of these elements we should also write M to Y. So you must define the operator space in which this love question is defined.

All right. So, that's the end of class. I know this has been a lot. Thank you for staying until the end. As always, the notebook that I've handed out is going to be your best friend. It's got lecture notes, supplemental notes in there. You can also always come to me at office hours, which I'll be holding tomorrow at the usual time, or you can email me with any questions that you have. Make sure, again, that you take a look at the problem set. It is due on Thursday, and we will be going over the problem sets on Thursday, as usual, and then you'll have your quiz next week as well.